Introducing Cosmology and General Relativity at high school level

Alice Gasparini & Andreas Müller

UniGE – Collège de Genève

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SwissMAP

The Mathematics of Physics

National Centre of Competence in Research

UNIVERSITÉ DE GENÈVE

FACULTÉ DES SCIENCES

Didactique de la physique

Motivation & Purposes

1. *Boost* the learners' level in mathematics and in traditionally taught physics

(e.g. classical mechanics, waves, electromagnetism or thermodynamics)

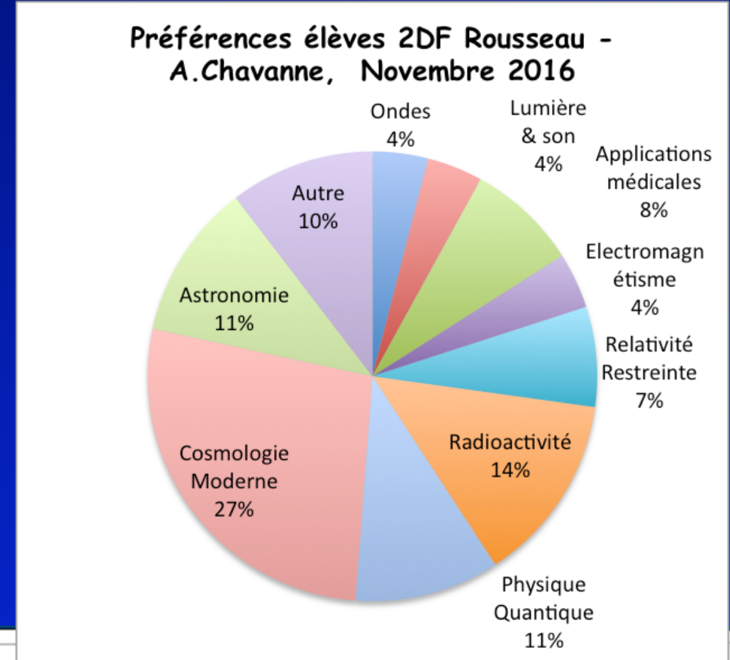
by attractive subjects :

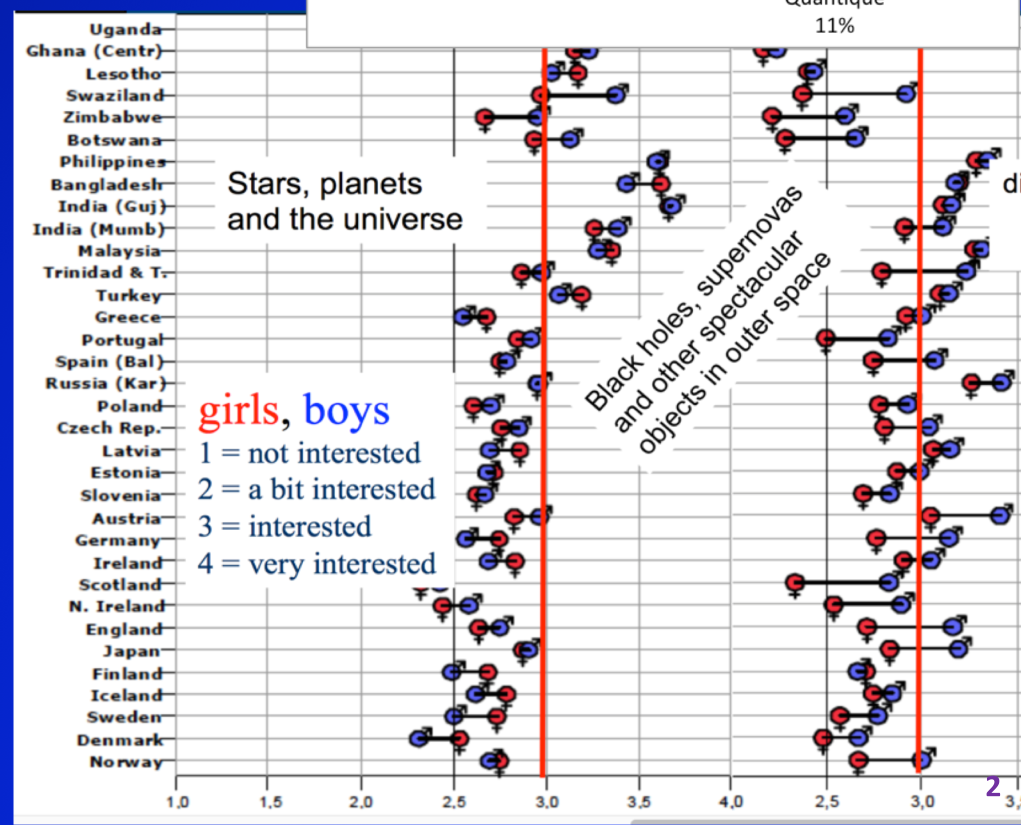
not *replace* but *complete* the "classical" curriculum

Research ROSE

(Relevance of Science Education)

- ➢ Similar results across many countries
- ➢ Averages for « ordinary » science subjects are about 2





Motivation & Purposes

2. *Improve* the links between high school and actual research

   Students who learn physics up to the 19th century have a distorted idea about the main issues of modern physics

3. *Medium level* of elementarisation

   Activities based on high school skills on maths and "classical" taught physics

   ➡ not for a wide public (zero equations), not academic level

- Contents are based exclusively on the sec II sciences' study plan ("gymnase")
- Thematic paths throughout the chapters with progressive level of difficulty

Teaching material

Course of 8 chapters

- theory (book, 2018 & 2023 in FR; 2024 in IT; EN planned for 2024-25)
- exercises & activities with corrections

https://nccr-swissmap.ch/school-teachers-children/general-relativity

https://physalice.ch/cosmologie/

Table des matières

→ Ideal for a physics optional course (2h/week), starting from senior high school

→ Toolbox for many others teaching contexts (punctual activities)

→ Complete intro for any interested learner with a good scientific background

Impact

1. Course built over 7 years in parallel with practice by more than 10 teachers and their class-groups in CH, FR, CA and BE
   → recognized by the international "Contemporary Physics Education Prize" (CPEP) in 2020

2. Quantitative data show positive effects on curiosity related to physics (effect size Cohen d = 0.4) and on conceptual understanding (d = 2.4)

3. Qualitative data (interviews) of students, stating: an increased motivation by the course; the comprehensibility, clear structure and organisation of the course; an enlarged understanding of what physics and of the nature of research are about
   e.g. "This course changed my vision of astronomy, I especially had stereotypes"; "I discovered the importance of math in physics"; "chose the PYOC option for this course"; "These are almost magical subjects…"; "these subjects should be integrated to the normal physics curriculum "

4. Courses with Gender Ratio (GR, girls : boys) ≥ 1 (for optional phy/maths courses GR ≤ 0.3)

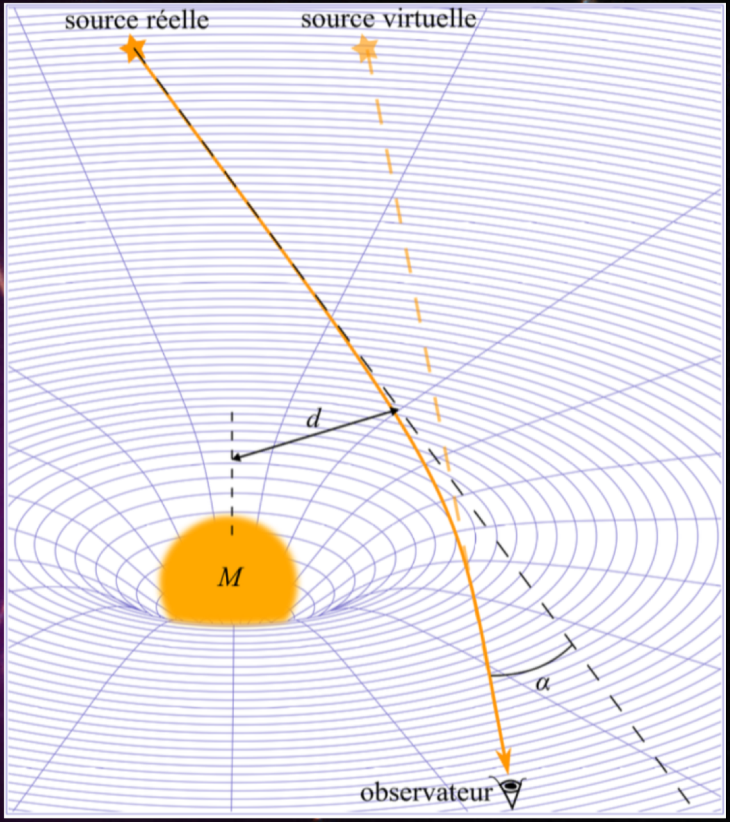
5. The content and structure of the course have been cross-evaluated and validated by university experts (RG, cosmology and didactics) and as part of the publication of the book and scientific bulletins (see e.g. SPS n. 55 or SSPMP n. 137)

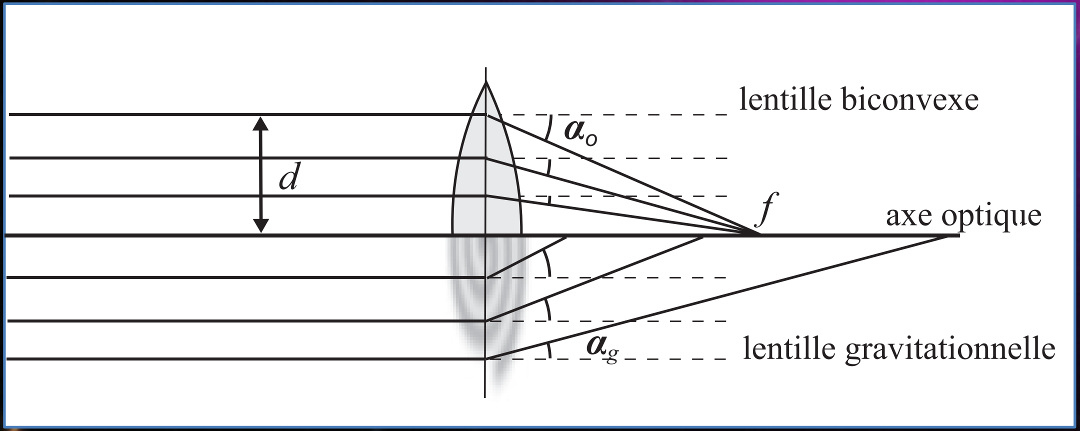
6. Evaluation of teachers during continuum professional formation (Geneva, Valais, Ticino): general opinion 3.9 / 4

Example of activities

$$\alpha \propto \frac{GM}{c^2 d}$$

- Deflection angle
  - With dimension analysis
  - Newtonian derivation (factor ½)





- How to demonstrate this 1/*d* behaviour ?

Example of activities

- A simple integration to find the profile of the corresponding optical lens profile

e) *Puisque i est l'angle entre le rayon incident (vertical) et la normale à la courbe $y(x)$ au point P, la tangente à cette courbe en ce point est $y'(x) = dy/dx = -i(x)$. Remplacer l'expression obtenue au point d) pour $i(x)$ puis intégrer cette équation pour trouver le profil $y(x)$.*

$$f'(x) \cong \frac{dy}{dx} = -i(x) = -\frac{4GM}{c^2(n-1)\cdot x}$$

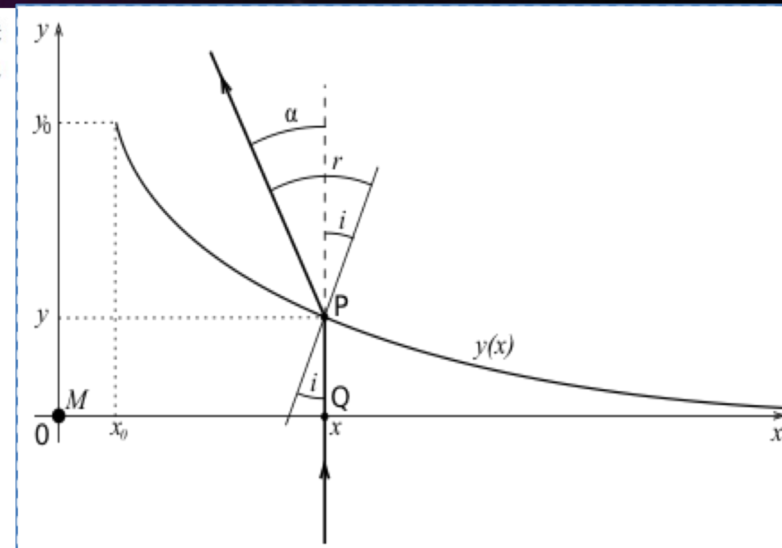
$$\Rightarrow \quad y(x) \cong -\frac{4GM}{c^2(n-1)} \cdot \ln\left(\frac{x}{x_0}\right) + y(x_0)$$

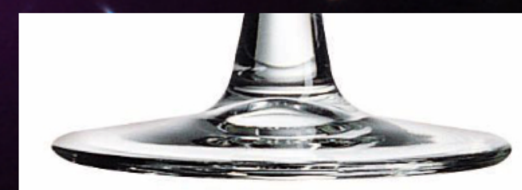
$$\Rightarrow \quad y(x) \cong \frac{4GM}{c^2(n-1)} \cdot \ln\left(\frac{x_0}{x}\right) + y_0$$

où $y_0 = y(x_0)$.

Donc le profil est proportionnel au logarithme de l'inverse de x :

$$y(x) \propto \ln\left(\frac{1}{x}\right) .$$

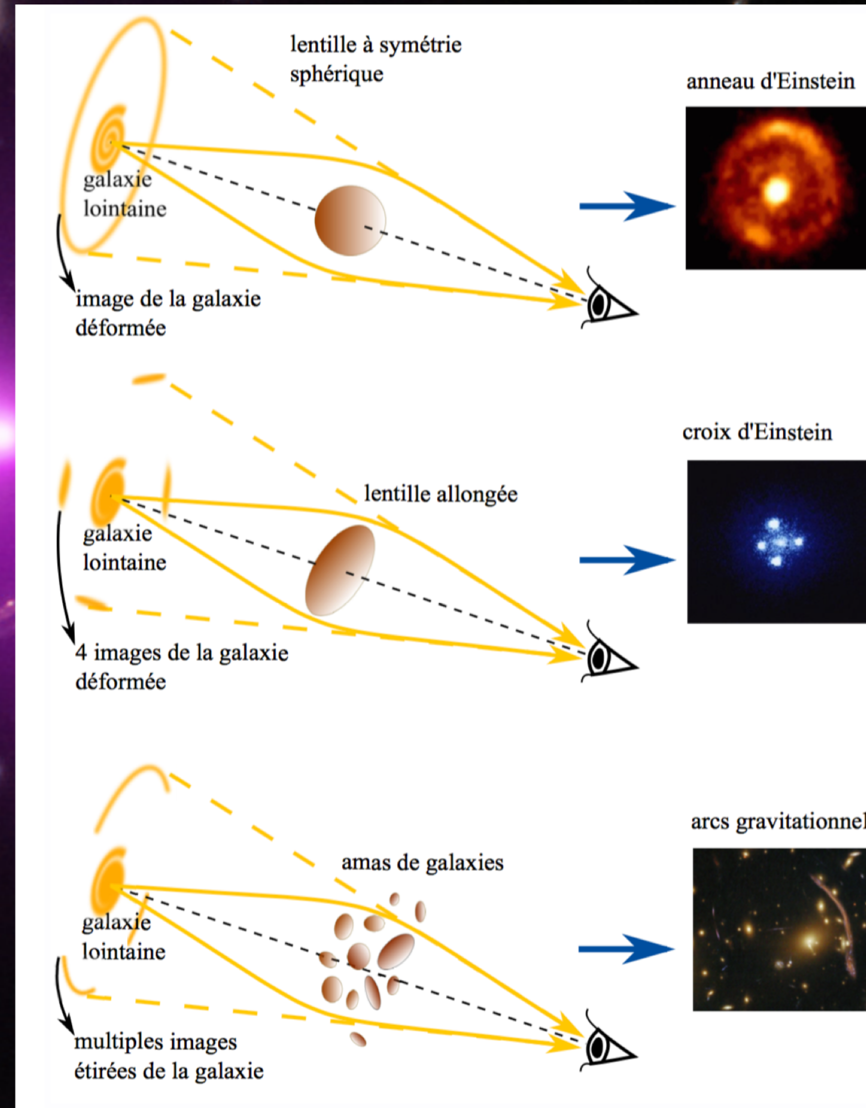




- Trigonometry + algebra to find *M* as a function of the θ_{Einst} and the distances

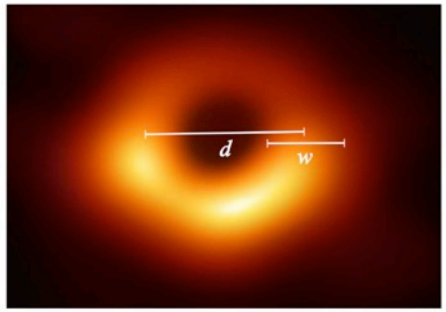
$$\theta_{Einst} = \sqrt{\frac{4GMD_{SL}}{c^2 D_{SO} D_{LO}}}$$

Example of activities



Example of activities

b) Expliquer pourquoi l'image du disque d'accrétion est un anneau, même si son axe de rotation ne pointe pas vers l'observateur. De quel phénomène s'agit-il ? (Faire un schéma si nécessaire.)



Crédit : Jifeng, L. *et al.*, Nature, Vol. 575, 618–621 (2019).

Pour les calculs qui suivent, utiliser les données ci-dessous :

Masse du trou noir : $M = 6,5 \cdot 10^9 M_\odot$

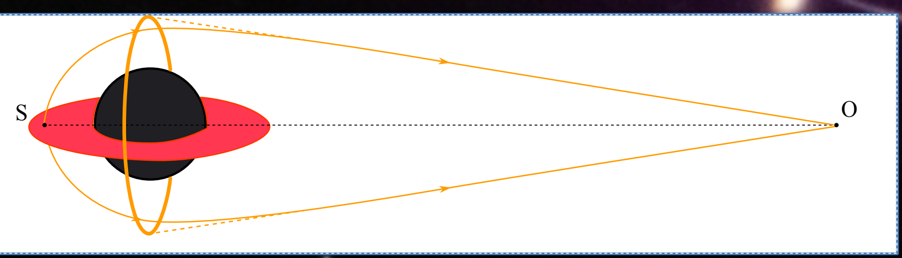
Distance entre la Terre et M87* : $D = 16,8$ Mpc

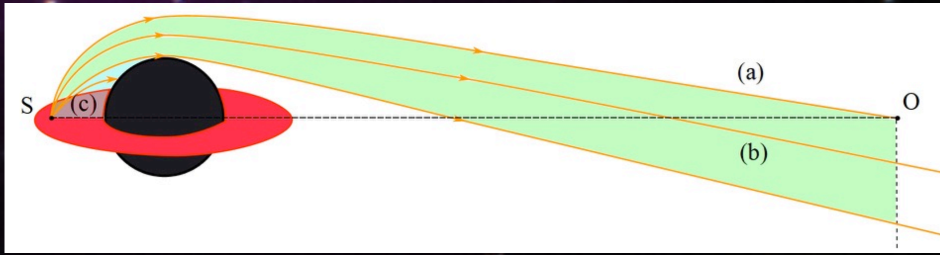
THE ASTROPHYSICAL JOURNAL LETTERS, 875:L1 (17pp), 2019 April 10

Table 1
Parameters of M87*

Parameter	Estimate
Ring diameter [a] d	$42 \pm 3\ \mu$as
Ring width [a]	$<20\ \mu$as
Crescent contrast [b]	$>10:1$
Axial ratio [a]	$<4:3$
Orientation PA	150°–200° east of north
$\theta_g = GM/Dc^2$ [c]	$3.8 \pm 0.4\ \mu$as
$\alpha = d/\theta_g$ [d]	$11^{+0.5}_{-0.3}$
M [c]	$(6.5 \pm 0.7) \times 10^9 M_\odot$
Parameter	**Prior Estimate**
D [e]	(16.8 ± 0.8) Mpc
M(stars) [e]	$6.2^{+1.1}_{-0.6} \times 10^9 M_\odot$
M(gas) [e]	$3.5^{+0.9}_{-0.3} \times 10^9 M_\odot$

Notes.
[a] Derived from the image domain.
[b] Derived from crescent model fitting.
[c] The mass and systematic errors are averages of the three methods (geometric models, GRMHD models, and image domain ring extraction).
[d] The exact value depends on the method used to extract d, which is reflected in the range given.
[e] Rederived from likelihood distributions (Paper VI).



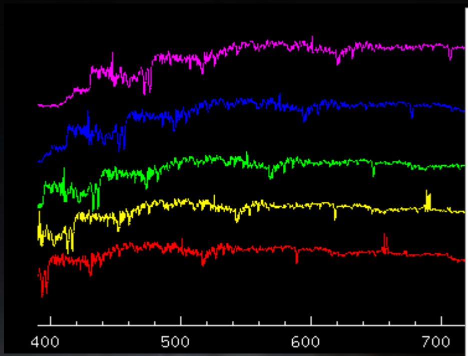


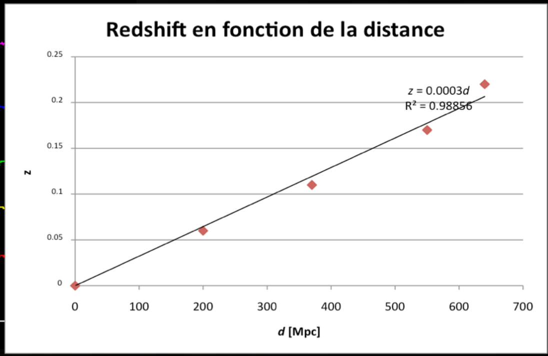
$$r_{\text{ombre}} = \theta_{\text{ombre}} \cdot D = 53 \cdot 10^{-12} \cdot 52 \cdot 10^{22} = 28 \cdot 10^{12}\ \text{m}\ >\ r_S$$

$$\Rightarrow \quad \frac{r_{\text{ombre}}}{r_S} = \frac{28 \cdot 10^{12}\ \text{m}}{19 \cdot 10^{12}\ \text{m}} = 1,5\ .$$

Example of activities

- Finding Hubble-Lemaître's law by comparing some nearby galaxies spectra

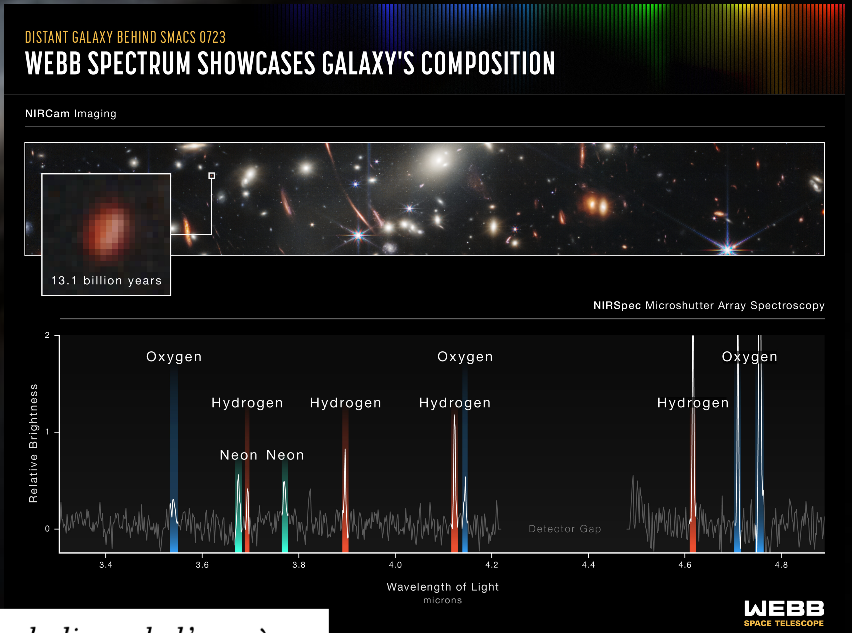




- Comparing the OOM of the expansion speed at different scales

H_0 = 70 km/s /Mpc = … /km = … /Gpc

- Difference between
  - ➢ Doppler effect and cosmological redshift
  - ➢ Hubble radius ($z < 1$) and horizon ($z < \infty$)



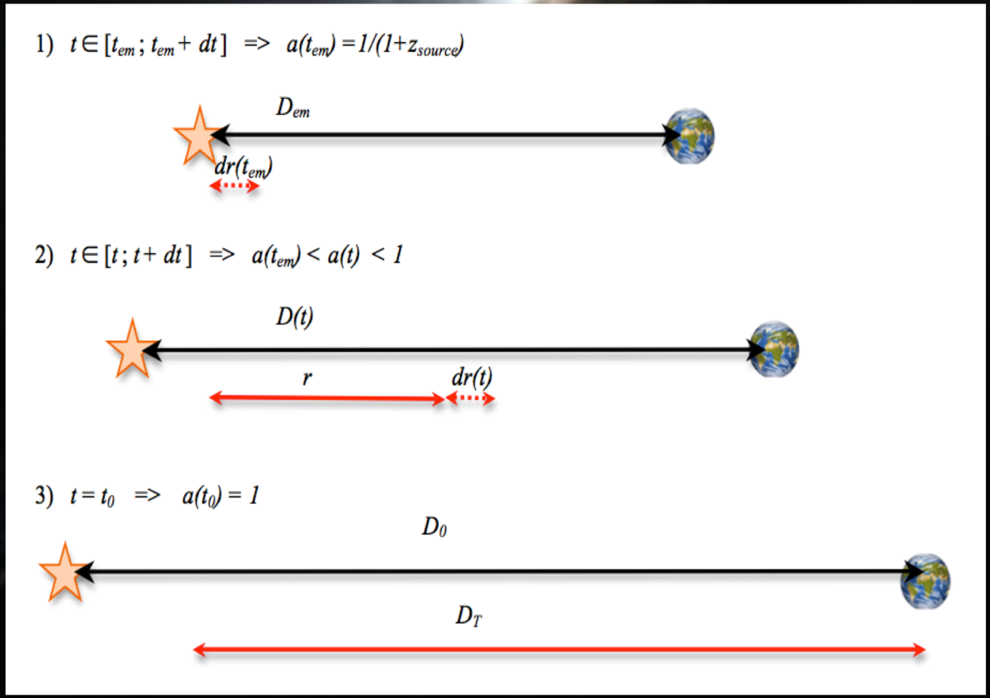
c) *En lisant dans le graphique la valeur de la longueur d'onde observée pour la ligne de l'oxygène OIII, émise il y a environ 13,1 milliards d'années par cette source lointaine, calculer son redshift.*

On peut lire sur le graphique $\lambda_0 = 4,75\,\mu$m et on sait que $\lambda = 0,5007\,\mu$m, donc

$$z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{4,75 - 0,5007}{0,5007} = 8,49\,.$$

Example of activities

- Finite value of c + expansion => the notion of distance splits into Proper/Comoving/Light-travel/Angular/Luminosity distance
  - ➢ finding the integral formula for each one as a function of z, Ω_m and Ω_Λ
  - ➢ limit cases
  - ➢ Integration + comparing with the Supernova Cosmology Project data



Perspectives

A network of collaborating teachers and researchers to develop

- New activities and class interventions of researchers in cosmology and GR in a frame of a Researcher-Student-Teacher-Partnership (RSTP) for science communication

- New collaborations for observations (Stellarium in Gornergrat, OFXB in Saint Luc)

- Teachers' continuum professional development formation

- Questioners and interviews to evaluate the impact of the course on students' learning and motivation

- Translation of the course material in Italian and English

https://nccr-swissmap.ch/school-teachers-children/general-relativity

https://physalice.ch/cosmologie/

Thank you

alice.gasparini@unige.ch

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More exemples of exercises & activities

- Compare gravitational interaction and electromagnetic interaction

    - For the system e^-/p^+ in the H atom, and for the system Moon/Earth (if we could get all the electrons out of them): $F_{em}/F_g \sim 10^{39}$

    - For $F_{em} \sim F_g$ between 2 identical particles, we need their $m/q \sim 10^{10}$ kg/C. Replacing $q = e$, we get $m \sim 10^{27}$eV (unification energy)

    - Equivalence principle and space-time curvature

- BH: Use conservation of mechanical energy to get the escape speed formula $v_l^2 = 2GM/R$

    - Schwarzschild Radius => when $v_l = c$

    - Compare v_l and thermal speed of different gases (H_2, N_2) to explain the composition of planets' atmosphere

- BH: Use Wien's law ***or*** the tunnel effect (Heisenberg indet. principle) to get its temperature + Estimate the time of evaporation using Stefan-Boltzmann law

More exemples of exercises & activities

- Fusion temperature from the nuclei's size and the Coulomb potential energy

- Why fusions starts at (much) lower temperature ?
  => Quantum Tunnelling

- What is the speed of a couple proton/antiproton to produce an Higgs Boson ?

- Quadrupole formula for the amplitude of a GW:

  - Comparing quadrupole (GW) and dipole (EMW) emission
  - Why only astrophysical masses can produce such a wave ?
  - Why do we need relativistic sources ? (BH or NS)
  - Why are these waves so important in the nowadays physics ?
  - Relation M_{source}/ frequency/ detector's size
  - + all the exercises you can do with waves (traditional curriculum)